Thermodynamics: An Engineering Approach, 6th Edition
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Chapter 7

ENTROPY

Clasius inequality

$$\oint \frac{\delta Q}{T} \leq 0$$

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int rev}}$$

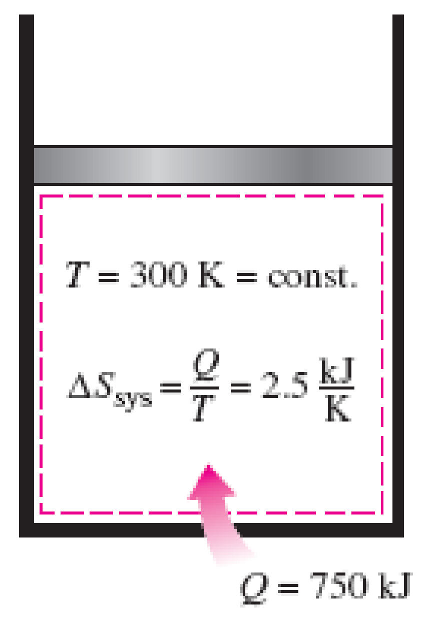
Isothermal Heat Transfer Processes

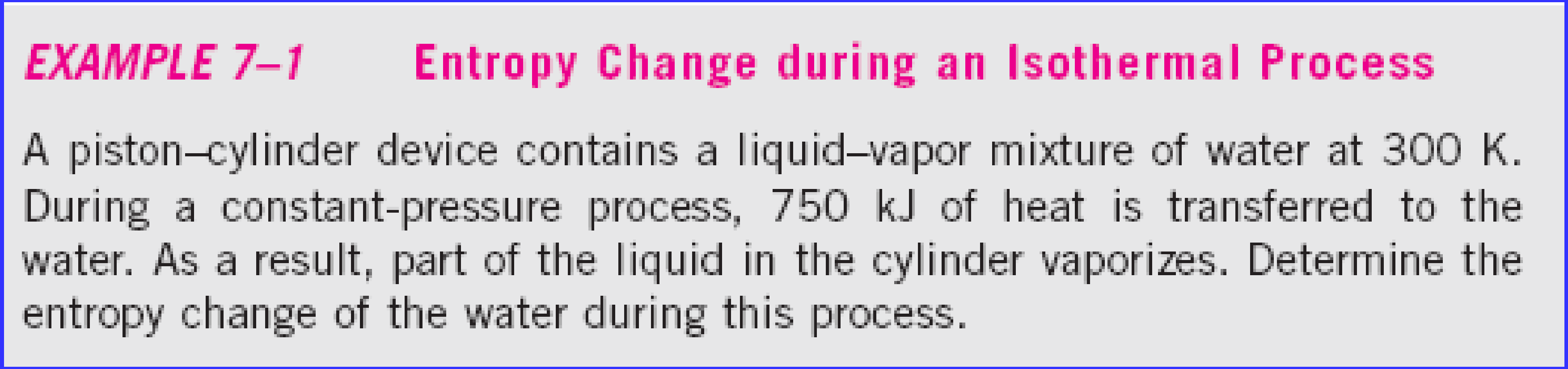
$$\Delta S = \frac{Q}{T_0}$$

The entropy generation *S*gen is always a *positive* quantity or zero

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0$$

Example (7.1)





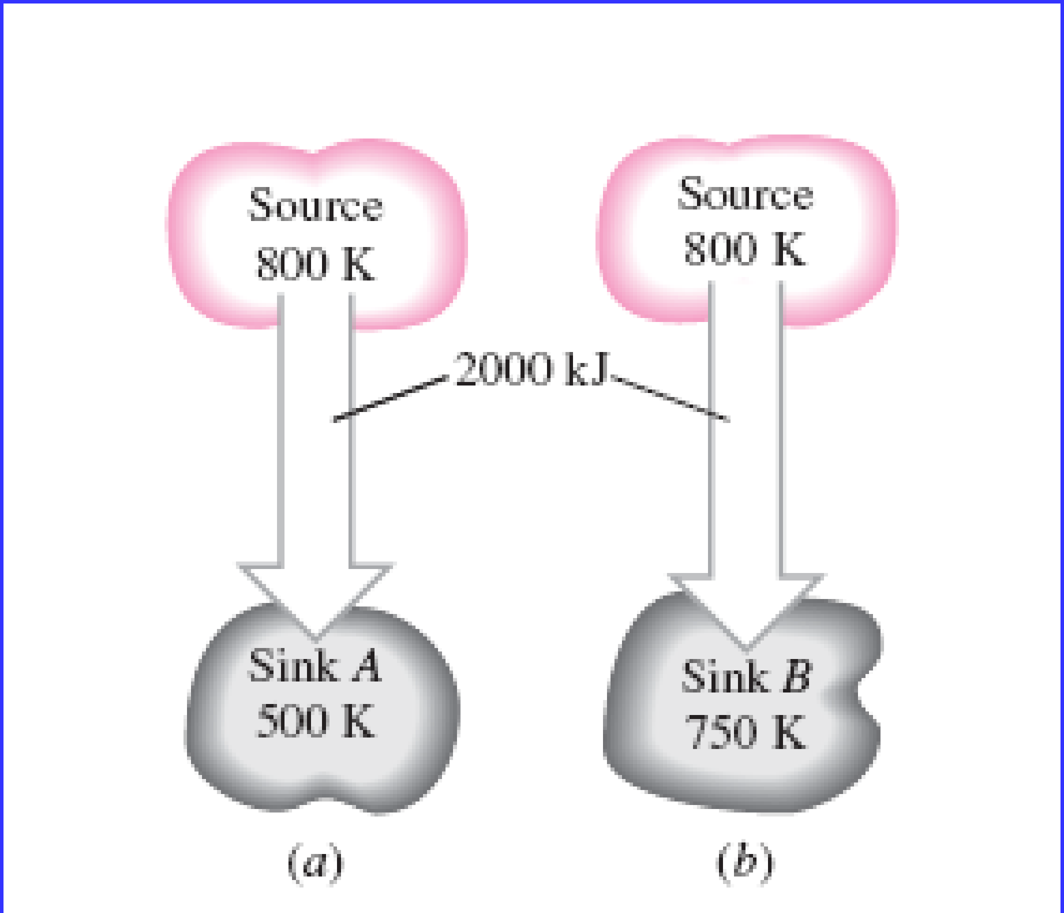
EXAMPLE 7–1 Entropy Change during an Isothermal Process

A piston–cylinder device contains a liquid–vapor mixture of water at 300 K. During a constant-pressure process, 750 kJ of heat is transferred to the water. As a result, part of the liquid in the cylinder vaporizes. Determine the entropy change of the water during this process.

$$\Delta S_{\text{sys,isothermal}} = \frac{Q}{T_{\text{sys}}} = \frac{750\ \text{kJ}}{300\ \text{K}} = 2.5\ \text{kJ/K}$$

EXAMPLE 7–2 **Entropy Generation during Heat Transfer Processes**

A heat source at 800 K loses 2000 kJ of heat to a sink at (*a*) 500 K and (*b*) 750 K. Determine which heat transfer process is more irreversible.



The entropy change for each reservoir can be determined from Eq. 7–6 since each reservoir undergoes an internally reversible, isothermal process.

(*a*) For the heat transfer process to a sink at 500 K:

$$\Delta S_{\text{source}} = \frac{Q_{\text{source}}}{T_{\text{source}}} = \frac{-2000\ \text{kJ}}{800\ \text{K}} = -2.5\ \text{kJ/K}$$

$$\Delta S_{\text{sink}} = \frac{Q_{\text{sink}}}{T_{\text{sink}}} = \frac{2000\ \text{kJ}}{500\ \text{K}} = +4.0\ \text{kJ/K}$$

and

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{source}} + \Delta S_{\text{sink}} = (-2.5 + 4.0)\ \text{kJ/K} = \mathbf{1.5\ kJ/K}$$

Therefore, 1.5 kJ/K of entropy is generated during this process. Noting that both reservoirs have undergone internally reversible processes, the entire entropy generation took place in the partition.

(*b*) Repeating the calculations in part (*a*) for a sink temperature of 750 K, we obtain

$$\Delta S_{\text{source}} = -2.5\ \text{kJ/k}$$

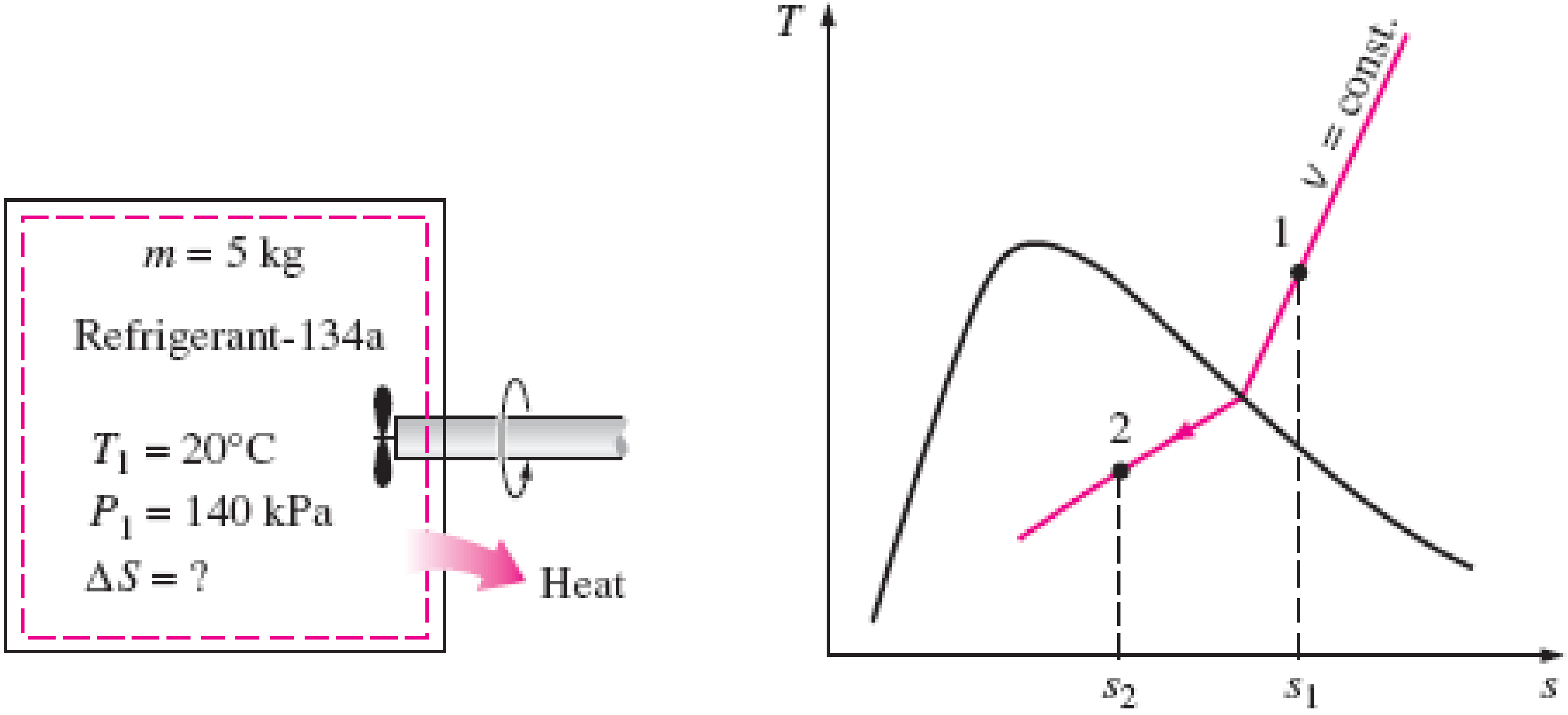
$$\Delta S_{\text{sink}} = +2.7\ \text{kJ/K}$$

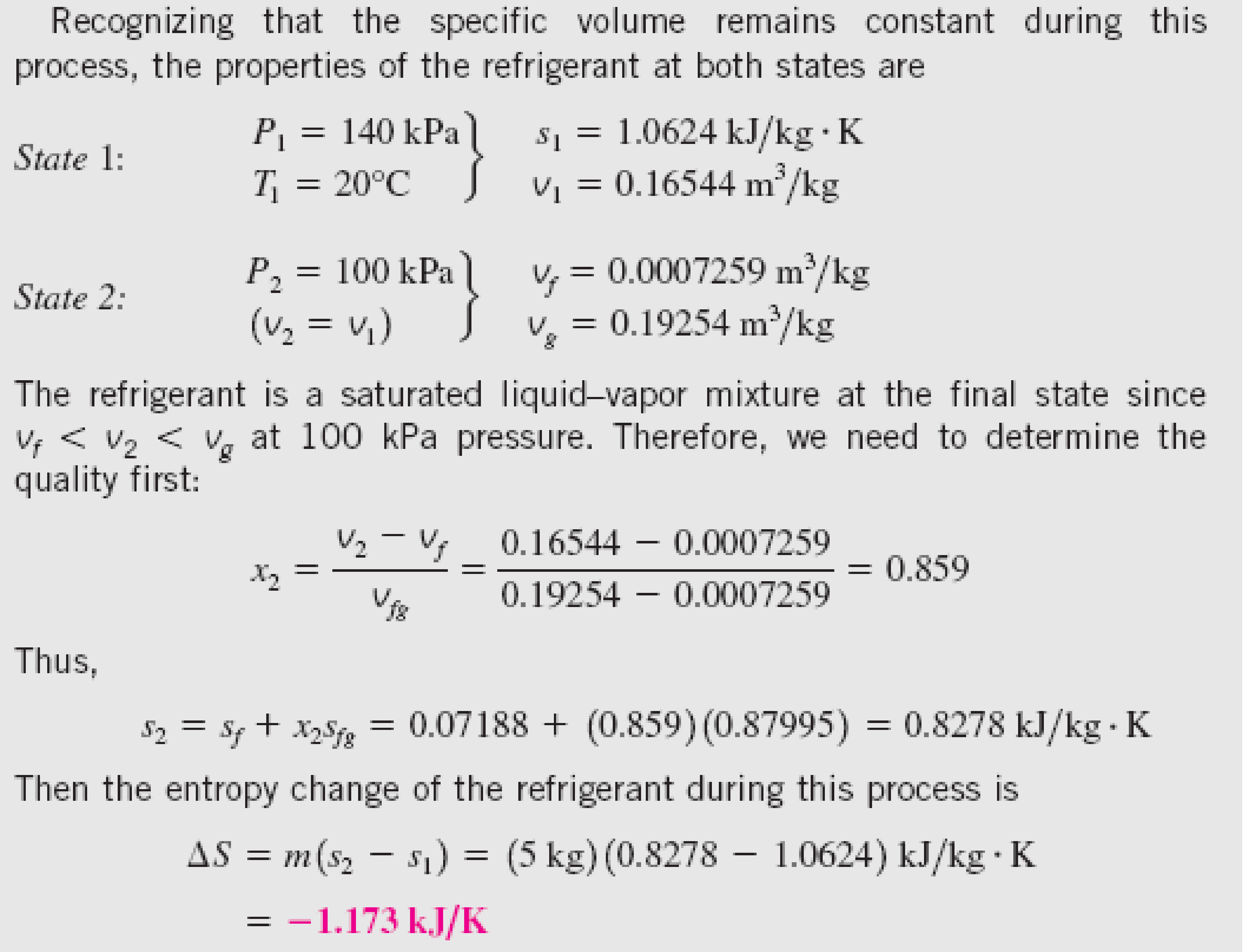
and

$$S_{\text{gen}} = \Delta S_{\text{total}} = (-2.5 + 2.7)\ \text{kJ/K} = \mathbf{0.2\ kJ/K}$$

EXAMPLE 7–3 Entropy Change of a Substance in a Tank

A rigid tank contains 5 kg of refrigerant-134a initially at 20°C and 140 kPa. The refrigerant is now cooled while being stirred until its pressure drops to 100 kPa. Determine the entropy change of the refrigerant during this process.





Recognizing that the specific volume remains constant during this process, the properties of the refrigerant at both states are

State 1:

$$\left.\begin{aligned} P_1 &= 140\ \text{kPa} \\ T_1 &= 20^\circ\text{C} \end{aligned}\right\} \quad \begin{aligned} s_1 &= 1.0624\ \text{kJ/kg}\cdot\text{K} \\ v_1 &= 0.16544\ \text{m}^3/\text{kg} \end{aligned}$$

State 2:

$$\left.\begin{aligned} P_2 &= 100\ \text{kPa} \\ (v_2 &= v_1) \end{aligned}\right\} \quad \begin{aligned} v_f &= 0.0007259\ \text{m}^3/\text{kg} \\ v_g &= 0.19254\ \text{m}^3/\text{kg} \end{aligned}$$

The refrigerant is a saturated liquid–vapor mixture at the final state since $v_f < v_2 < v_g$ at 100 kPa pressure. Therefore, we need to determine the quality first:

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.16544 - 0.0007259}{0.19254 - 0.0007259} = 0.859$$

Thus,

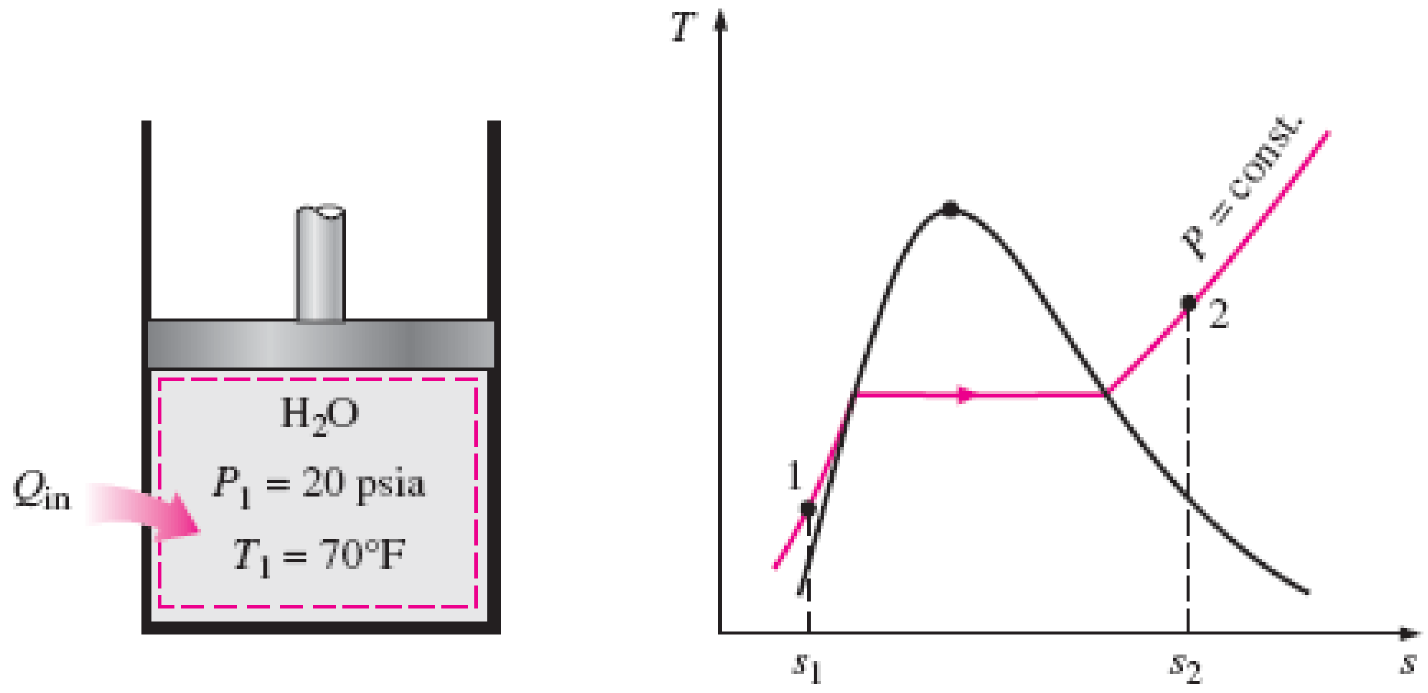
$$s_2 = s_f + x_2 s_{fg} = 0.07188 + (0.859)(0.87995) = 0.8278\ \text{kJ/kg}\cdot\text{K}$$

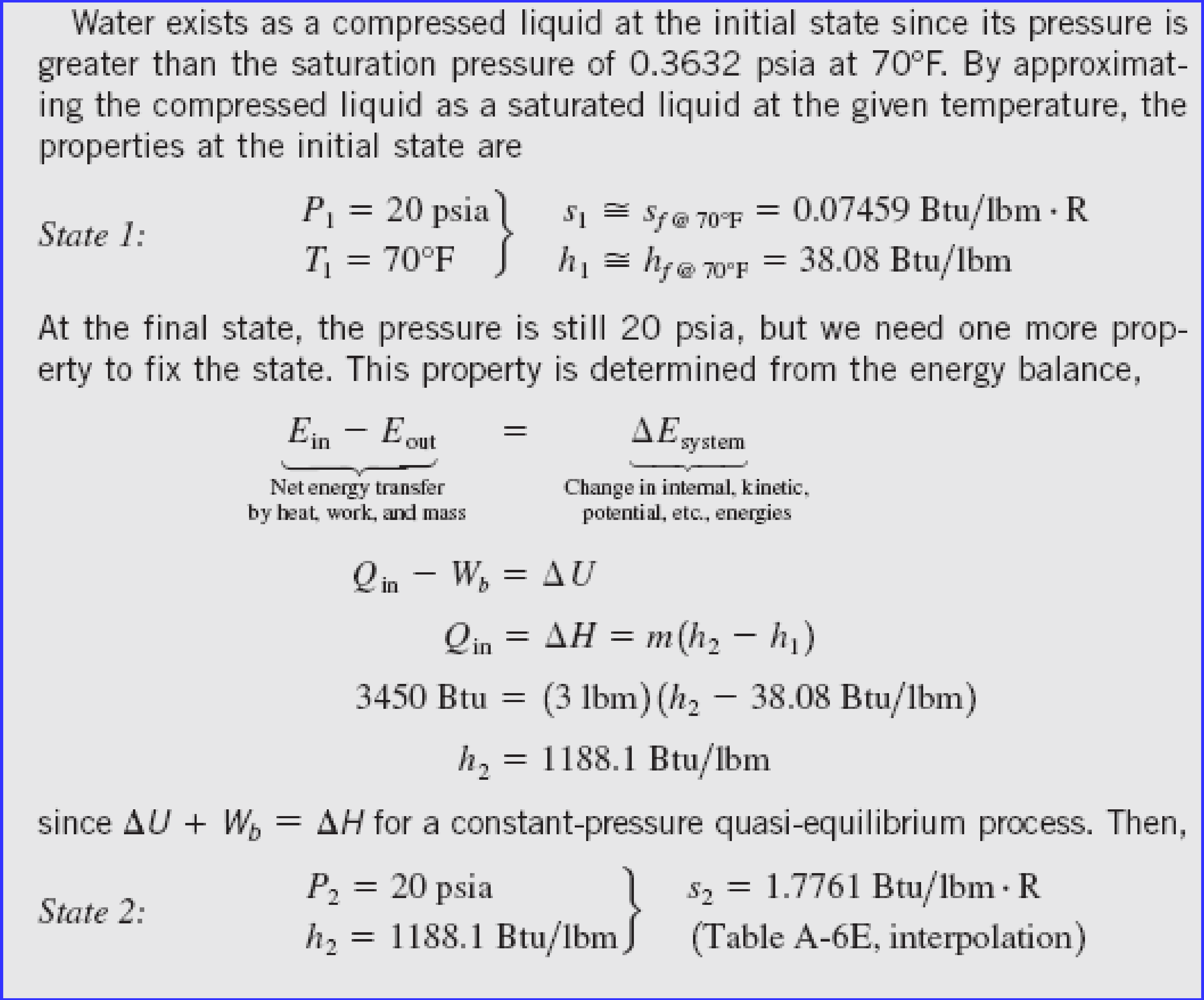
Then the entropy change of the refrigerant during this process is

$$\begin{aligned} \Delta S &= m(s_2 - s_1) = (5\ \text{kg})(0.8278 - 1.0624)\ \text{kJ/kg}\cdot\text{K} \\ &= \mathbf{-1.173\ kJ/K} \end{aligned}$$

EXAMPLE 7–4 Entropy Change during a Constant-Pressure Process

A piston–cylinder device initially contains 3 lbm of liquid water at 20 psia and 70°F. The water is now heated at constant pressure by the addition of 3450 Btu of heat. Determine the entropy change of the water during this process.





Water exists as a compressed liquid at the initial state since its pressure is greater than the saturation pressure of 0.3632 psia at 70°F. By approximating the compressed liquid as a saturated liquid at the given temperature, the properties at the initial state are

State 1:

$$\left.\begin{aligned} P_1 &= 20\text{ psia} \\ T_1 &= 70°\text{F} \end{aligned}\right\} \quad \begin{aligned} s_1 &\cong s_{f\,@\,70°\text{F}} = 0.07459\text{ Btu/lbm}\cdot\text{R} \\ h_1 &\cong h_{f\,@\,70°\text{F}} = 38.08\text{ Btu/lbm} \end{aligned}$$

At the final state, the pressure is still 20 psia, but we need one more property to fix the state. This property is determined from the energy balance,

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$Q_{\text{in}} - W_b = \Delta U$$

$$Q_{\text{in}} = \Delta H = m(h_2 - h_1)$$

$$3450\text{ Btu} = (3\text{ lbm})(h_2 - 38.08\text{ Btu/lbm})$$

$$h_2 = 1188.1\text{ Btu/lbm}$$

since $\Delta U + W_b = \Delta H$ for a constant-pressure quasi-equilibrium process. Then,

State 2:

$$\left.\begin{aligned} P_2 &= 20\text{ psia} \\ h_2 &= 1188.1\text{ Btu/lbm} \end{aligned}\right\} \quad \begin{aligned} &s_2 = 1.7761\text{ Btu/lbm}\cdot\text{R} \\ &(\text{Table A-6E, interpolation}) \end{aligned}$$

$$S_{\text{gen}} \begin{cases} > 0 & \text{Irreversible process} \\ = 0 & \text{Reversible process} \\ < 0 & \text{Impossible process} \end{cases}$$

Isentropic process

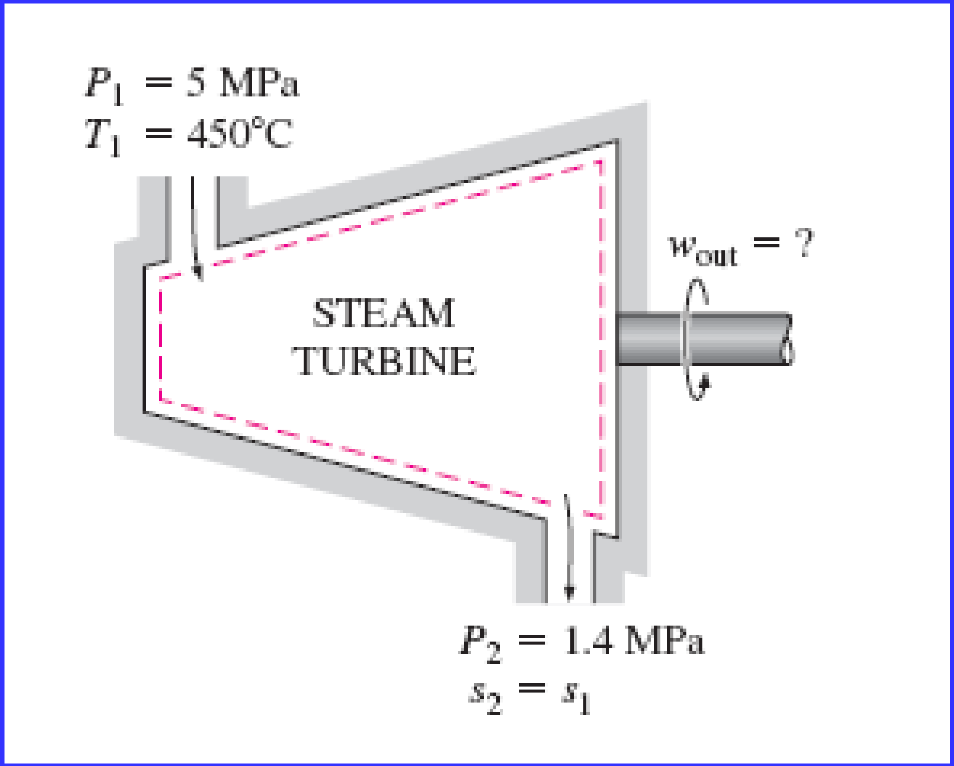
$$\Delta s = 0 \quad \text{or} \quad s_2 = s_1$$

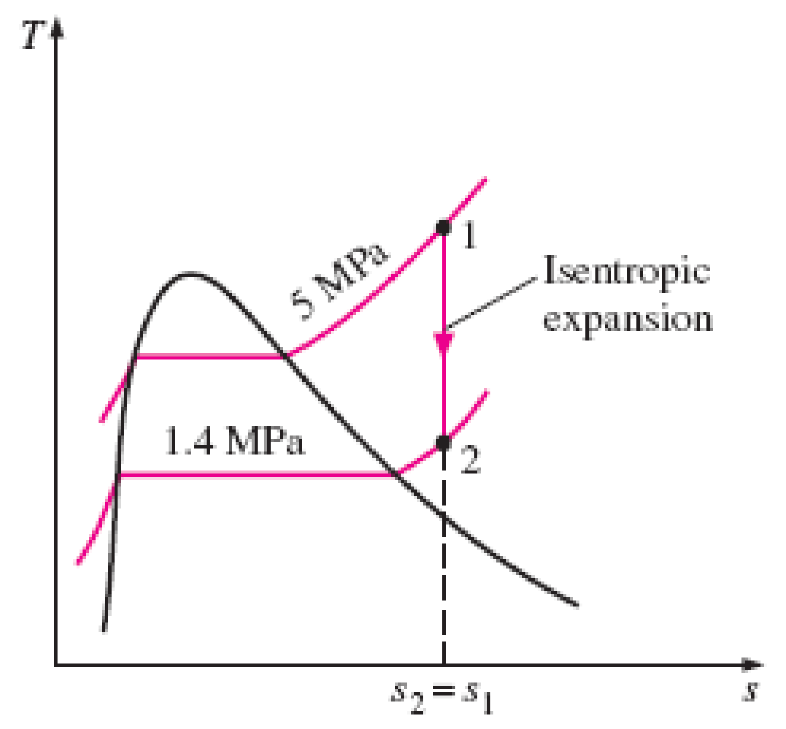
$$Q_{\text{int rev}} = \int_1^2 T\, dS$$

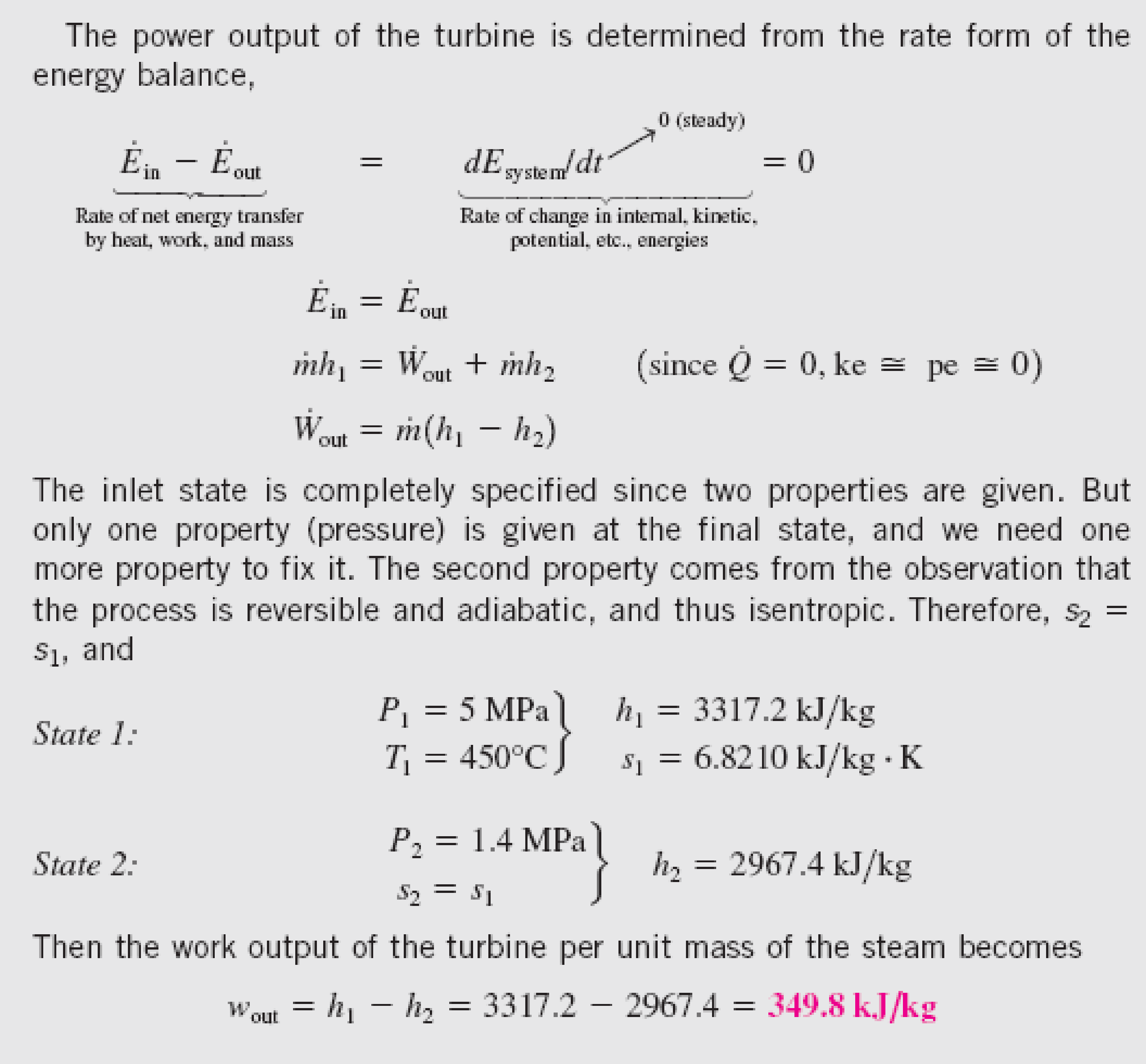
$$Q_{\text{int rev}} = T_0\, \Delta S$$

EXAMPLE 7–5 Isentropic Expansion of Steam in a Turbine

Steam enters an adiabatic turbine at 5 MPa and 450°C and leaves at a pressure of 1.4 MPa. Determine the work output of the turbine per unit mass of steam if the process is reversible.







The power output of the turbine is determined from the rate form of the energy balance,

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}h_1 = \dot{W}_{\text{out}} + \dot{m}h_2 \qquad (\text{since } \dot{Q} = 0, \text{ke} \cong \text{pe} \cong 0)$$

$$\dot{W}_{\text{out}} = \dot{m}(h_1 - h_2)$$

The inlet state is completely specified since two properties are given. But only one property (pressure) is given at the final state, and we need one more property to fix it. The second property comes from the observation that the process is reversible and adiabatic, and thus isentropic. Therefore, $s_2 = s_1$, and

State 1:

$$\left.\begin{array}{l} P_1 = 5 \text{ MPa} \\ T_1 = 450^\circ\text{C} \end{array}\right\} \begin{array}{l} h_1 = 3317.2 \text{ kJ/kg} \\ s_1 = 6.8210 \text{ kJ/kg}\cdot\text{K} \end{array}$$

State 2:

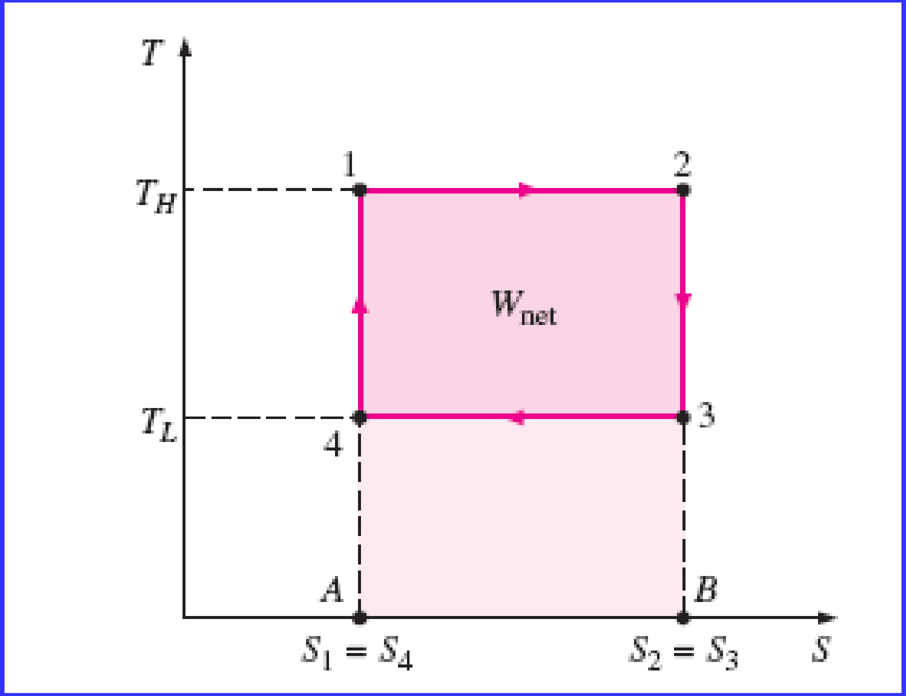
$$\left.\begin{array}{l} P_2 = 1.4 \text{ MPa} \\ s_2 = s_1 \end{array}\right\} h_2 = 2967.4 \text{ kJ/kg}$$

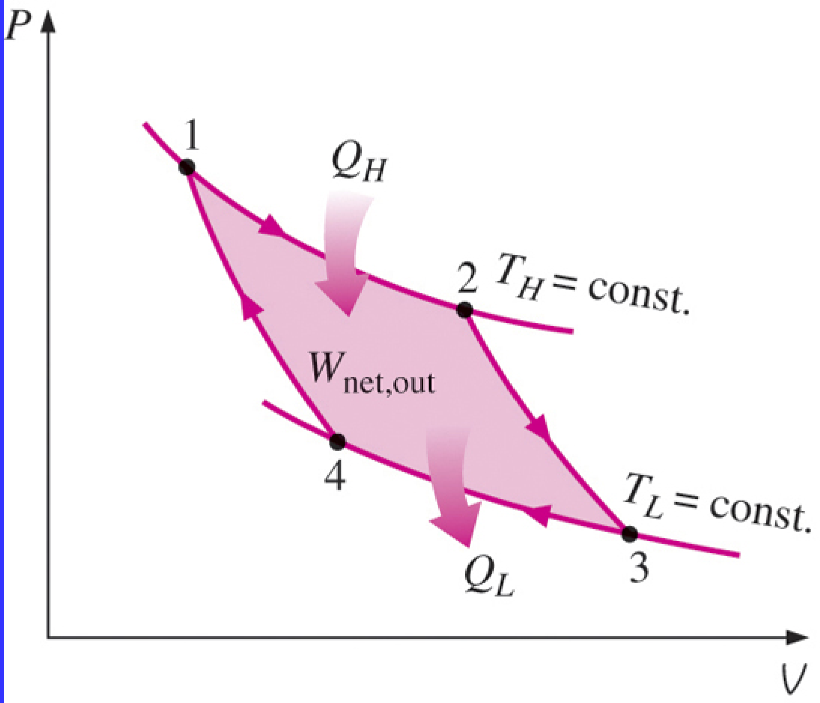
Then the work output of the turbine per unit mass of the steam becomes

$$w_{\text{out}} = h_1 - h_2 = 3317.2 - 2967.4 = \mathbf{349.8 \text{ kJ/kg}}$$

EXAMPLE 7–6 The *T-S* Diagram of the Carnot Cycle

Show the Carnot cycle on a *T-S* diagram and indicate the areas that represent the heat supplied Q_H, heat rejected Q_L, and the net work output $W_{net,out}$ on this diagram.





Solution The Carnot cycle is to be shown on a *T-S* diagram, and the areas that represent Q_H, Q_L, and $W_{\text{net,out}}$ are to be indicated.

Analysis Recall that the Carnot cycle is made up of two reversible isothermal (T = constant) processes and two isentropic (s = constant) processes. These four processes form a rectangle on a *T-S* diagram, as shown in Fig. 7–19.

On a *T-S* diagram, the area under the process curve represents the heat transfer for that process. Thus the area *A*12*B* represents Q_H, the area *A*43*B* represents Q_L, and the difference between these two (the area in color) represents the net work since

$$W_{\text{net,out}} = Q_H - Q_L$$

Therefore, the area enclosed by the path of a cycle (area 1234) on a *T-S* diagram represents the net work. Recall that the area enclosed by the path of a cycle also represents the net work on a *P-V* diagram.

The first *T ds*, or *Gibbs equation*

$$ds = \frac{du}{T} + \frac{P\,dv}{T}$$

The second *T ds equation*

$$ds = \frac{dh}{T} - \frac{v\,dP}{T}$$

$$\textit{Liquids, solids:} \qquad s_2 - s_1 = \int_1^2 c(T)\,\frac{dT}{T} \cong c_{\text{avg}} \ln \frac{T_2}{T_1} \qquad (\text{kJ/kg}\cdot\text{K})$$

$$\textit{Isentropic:} \qquad s_2 - s_1 = c_{\text{avg}} \ln \frac{T_2}{T_1} = 0 \quad \rightarrow \quad T_2 = T_1$$

THE ENTROPY CHANGE OF IDEAL GASES

$$s_2 - s_1 = \int_1^2 c_v(T)\,\frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

$$s_2 - s_1 = \int_1^2 c_p(T)\,\frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

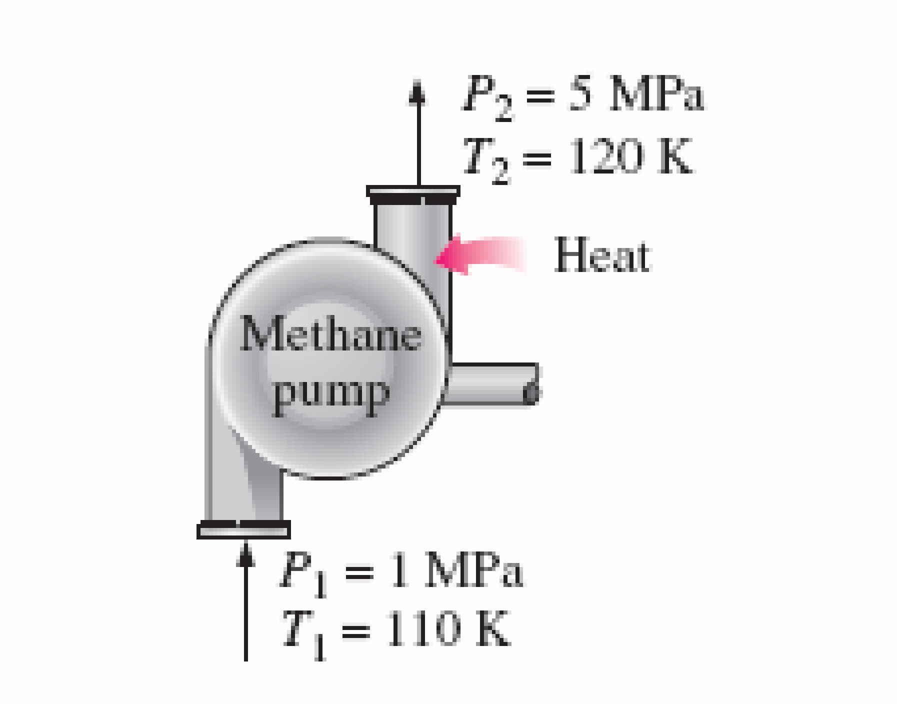
Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$s_2 - s_1 = c_{p,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

EXAMPLE 7–7 Effect of Density of a Liquid on Entropy

Liquid methane is commonly used in various cryogenic applications. The critical temperature of methane is 191 K (or −82°C), and thus methane must be maintained below 191 K to keep it in liquid phase. The properties of liquid methane at various temperatures and pressures are given in Table 7–1. Determine the entropy change of liquid methane as it undergoes a process from 110 K and 1 MPa to 120 K and 5 MPa (*a*) using tabulated properties and (*b*) approximating liquid methane as an incompressible substance. What is the error involved in the latter case?



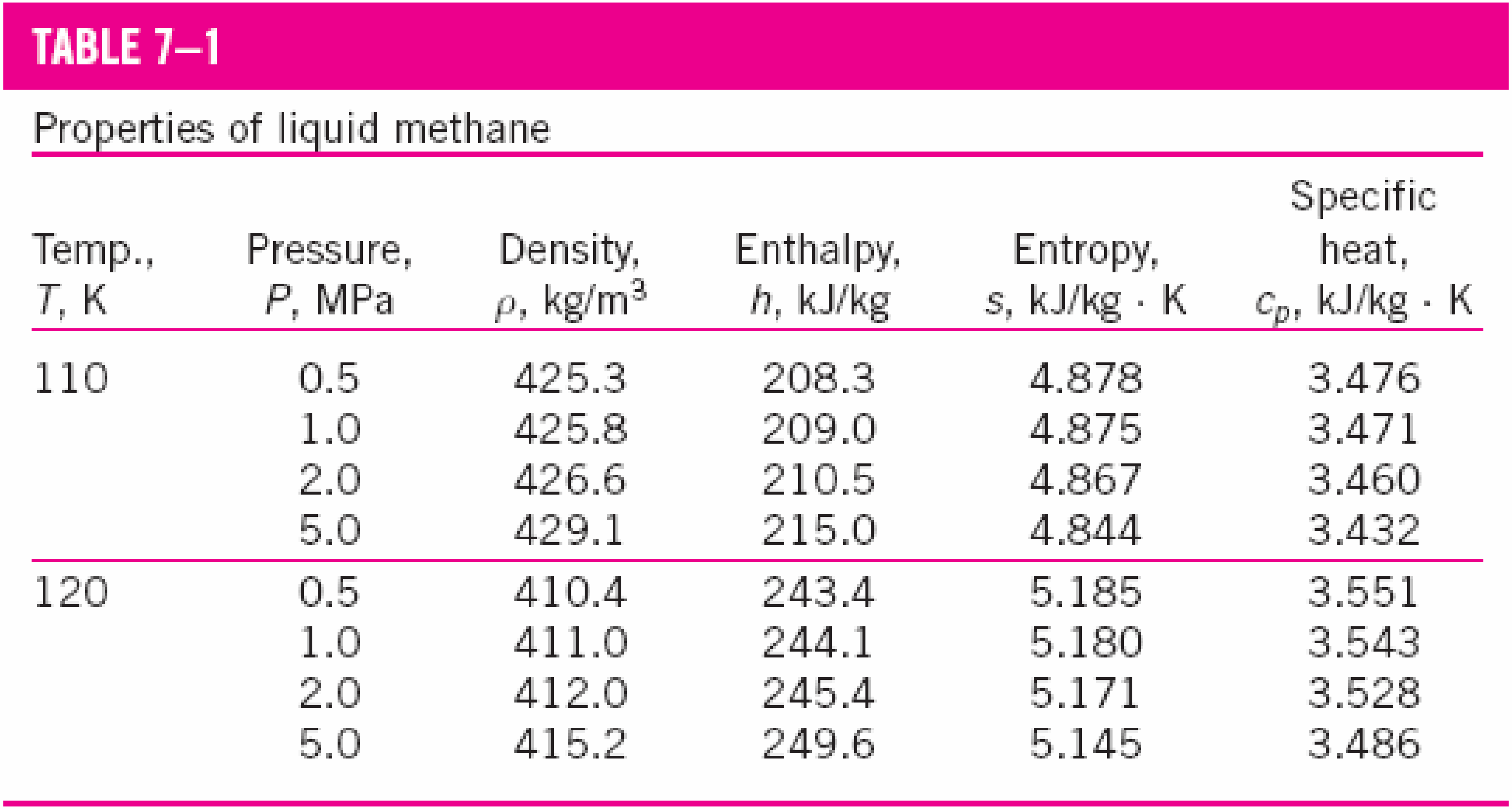


TABLE 7–1

Properties of liquid methane

Temp., T, K	Pressure, P, MPa	Density, ρ, kg/m^3	Enthalpy, h, kJ/kg	Entropy, s, kJ/kg · K	Specific heat, c_p, kJ/kg · K
110	0.5	425.3	208.3	4.878	3.476
	1.0	425.8	209.0	4.875	3.471
	2.0	426.6	210.5	4.867	3.460
	5.0	429.1	215.0	4.844	3.432
120	0.5	410.4	243.4	5.185	3.551
	1.0	411.0	244.1	5.180	3.543
	2.0	412.0	245.4	5.171	3.528
	5.0	415.2	249.6	5.145	3.486

State 1: $\left.\begin{aligned} P_1 &= 1 \text{ MPa} \\ T_1 &= 110 \text{ K} \end{aligned}\right\} \quad \begin{aligned} s_1 &= 4.875 \text{ kJ/kg} \cdot \text{K} \\ c_{p1} &= 3.471 \text{ kJ/kg} \cdot \text{K} \end{aligned}$

State 2: $\left.\begin{aligned} P_2 &= 5 \text{ MPa} \\ T_2 &= 120 \text{ K} \end{aligned}\right\} \quad \begin{aligned} s_2 &= 5.145 \text{ kJ/kg} \cdot \text{K} \\ c_{p2} &= 3.486 \text{ kJ/kg} \cdot \text{K} \end{aligned}$

Therefore,

$$\Delta s = s_2 - s_1 = 5.145 - 4.875 = \mathbf{0.270 \text{ kJ/kg} \cdot \text{K}}$$

(*b*) Approximating liquid methane as an incompressible substance, its entropy change is determined to be

$$\Delta s = c_{\text{avg}} \ln \frac{T_2}{T_1} = (3.4785 \text{ kJ/kg} \cdot \text{K}) \ln \frac{120 \text{ K}}{110 \text{ K}} = \mathbf{0.303 \text{ kJ/kg} \cdot \text{K}}$$

since

$$c_{\text{avg}} = \frac{c_{p1} + c_{p2}}{2} = \frac{3.471 + 3.486}{2} = 3.4785 \text{ kJ/kg} \cdot \text{K}$$

$$\text{Error} = \frac{|\Delta s_{\text{actual}} - \Delta s_{\text{ideal}}|}{\Delta s_{\text{actual}}} = \frac{|0.270 - 0.303|}{0.270} = \mathbf{0.122 \text{ (or } 12.2\%)}$$

Variable Specific Heats (Exact Analysis)

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} \qquad (\mathrm{kJ/kg \cdot K})$$

Isentropic Processes of Ideal Gases

Constant Specific Heats (Approximate Analysis)

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$\left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \left(\frac{v_1}{v_2}\right)^{k}$$

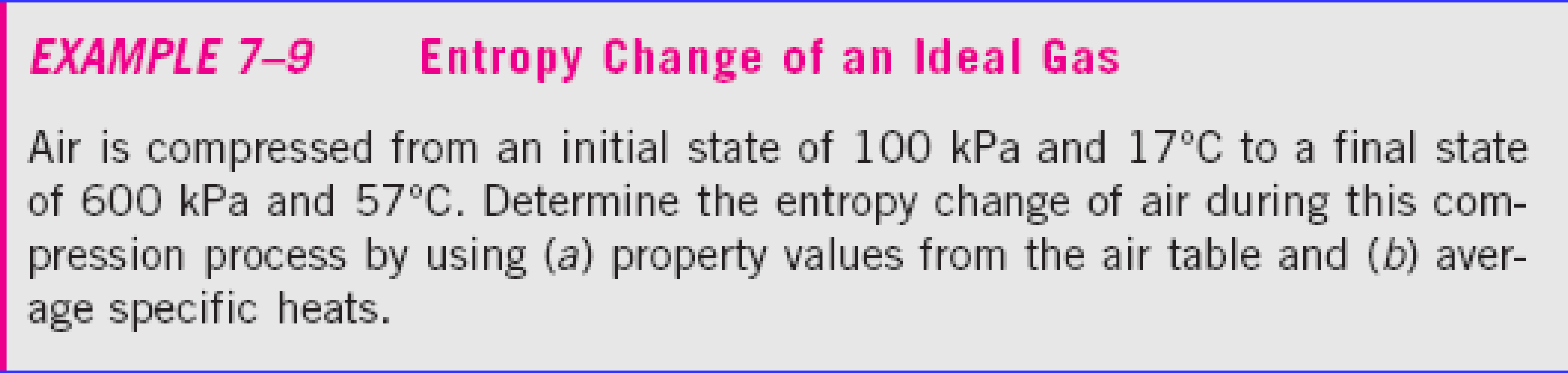
$$\begin{aligned} Tv^{k-1} &= \text{constant} \\ TP^{(1-k)/k} &= \text{constant} \\ Pv^{k} &= \text{constant} \end{aligned}$$

Isentropic Processes of Ideal Gases

Variable Specific Heats (Exact Analysis)

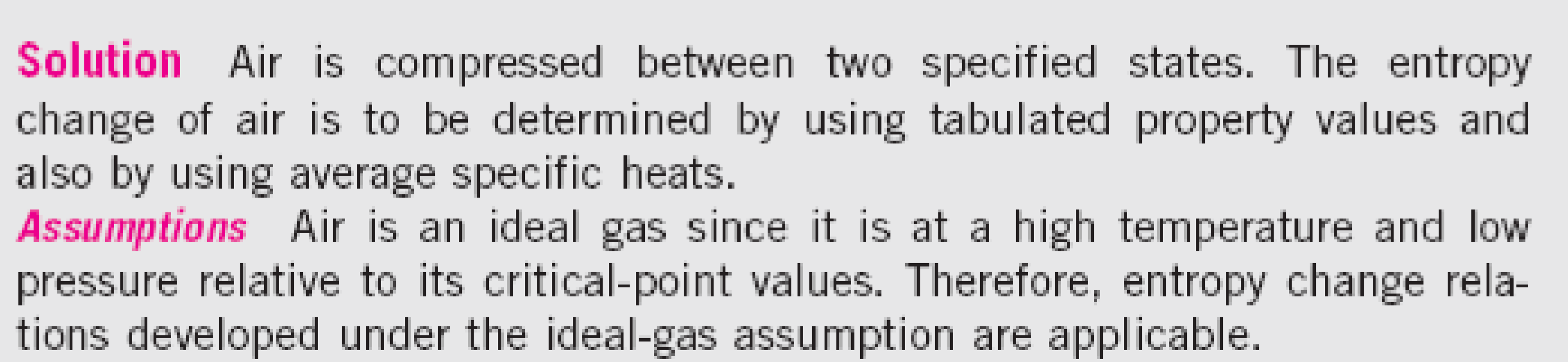
$$\left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \frac{P_{r2}}{P_{r1}}$$

$$\left(\frac{v_2}{v_1}\right)_{s=\text{const.}} = \frac{v_{r2}}{v_{r1}}$$



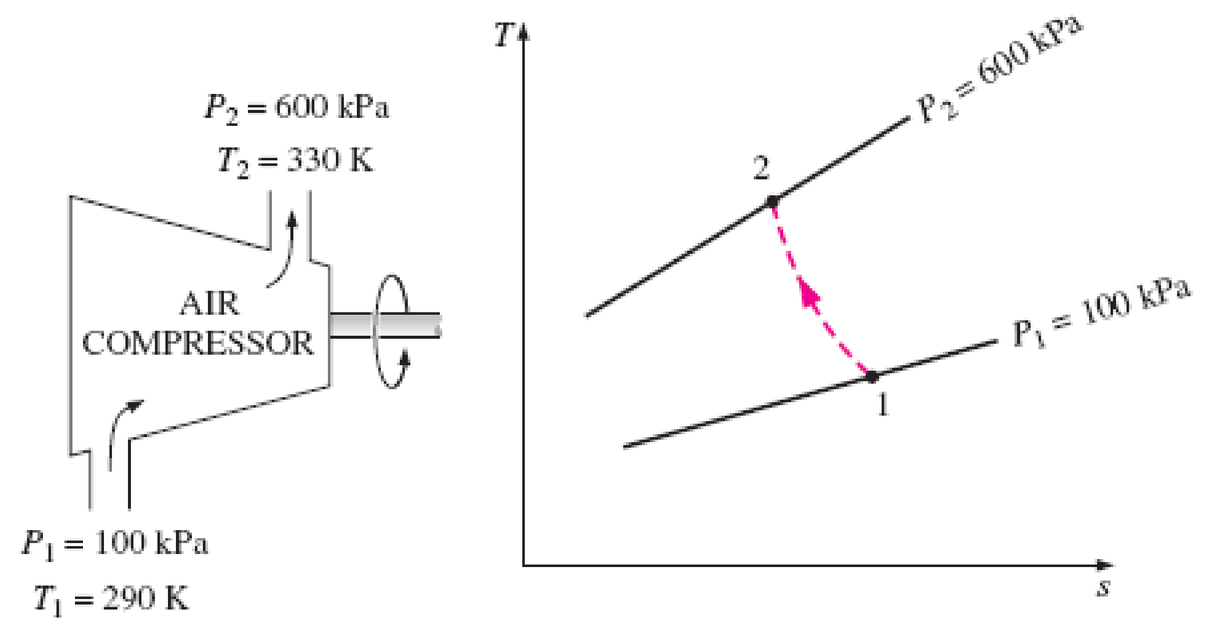
EXAMPLE 7–9 Entropy Change of an Ideal Gas

Air is compressed from an initial state of 100 kPa and 17°C to a final state of 600 kPa and 57°C. Determine the entropy change of air during this compression process by using (*a*) property values from the air table and (*b*) average specific heats.



Solution Air is compressed between two specified states. The entropy change of air is to be determined by using tabulated property values and also by using average specific heats.

Assumptions Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. Therefore, entropy change relations developed under the ideal-gas assumption are applicable.



Analysis A sketch of the system and the *T-s* diagram for the process are given in Fig. 7–34. We note that both the initial and the final states of air are completely specified.

(*a*) The properties of air are given in the air table (Table A–17). Reading $s°$ values at given temperatures and substituting, we find

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1}$$

$$= [(1.79783 - 1.66802)\ \text{kJ/kg} \cdot \text{K}] - (0.287\ \text{kJ/kg} \cdot \text{K}) \ln \frac{600\ \text{kPa}}{100\ \text{kPa}}$$

$$= \mathbf{-0.3844\ kJ/kg \cdot K}$$

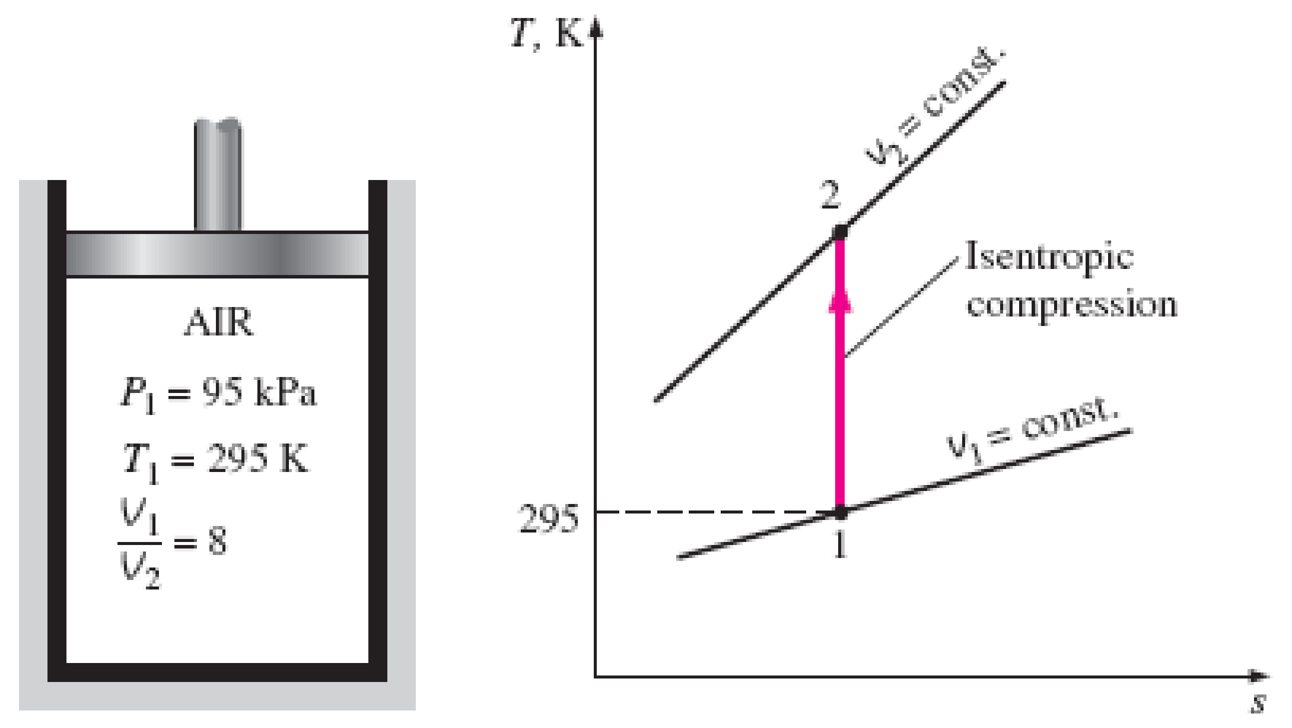
(*b*) The entropy change of air during this process can also be determined approximately from Eq. 7–34 by using a c_p value at the average temperature of 37°C (Table A–2b) and treating it as a constant:

$$
\begin{aligned}
s_2 - s_1 &= c_{p,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \\
&= (1.006 \text{ kJ/kg} \cdot \text{K}) \ln \frac{330 \text{ K}}{290 \text{ K}} - (0.287 \text{ kJ/kg} \cdot \text{K}) \ln \frac{600 \text{ kPa}}{100 \text{ kPa}} \\
&= \mathbf{-0.3842 \text{ kJ/kg} \cdot \text{K}}
\end{aligned}
$$

Discussion The two results above are almost identical since the change in temperature during this process is relatively small (Fig. 7–35). When the temperature change is large, however, they may differ significantly. For those cases, Eq. 7–39 should be used instead of Eq. 7–34 since it accounts for the variation of specific heats with temperature.

EXAMPLE 7–10 Isentropic Compression of Air in a Car Engine

Air is compressed in a car engine from 22°C and 95 kPa in a reversible and adiabatic manner. If the compression ratio V_1/V_2 of this engine is 8, determine the final temperature of the air.



This process is easily recognized as being isentropic since it is both reversible and adiabatic. The final temperature for this isentropic process can be determined from Eq. 7–50 with the help of relative specific volume data (Table A–17), as illustrated in Fig. 7–39.

For closed systems: $$\frac{V_2}{V_1} = \frac{v_2}{v_1}$$

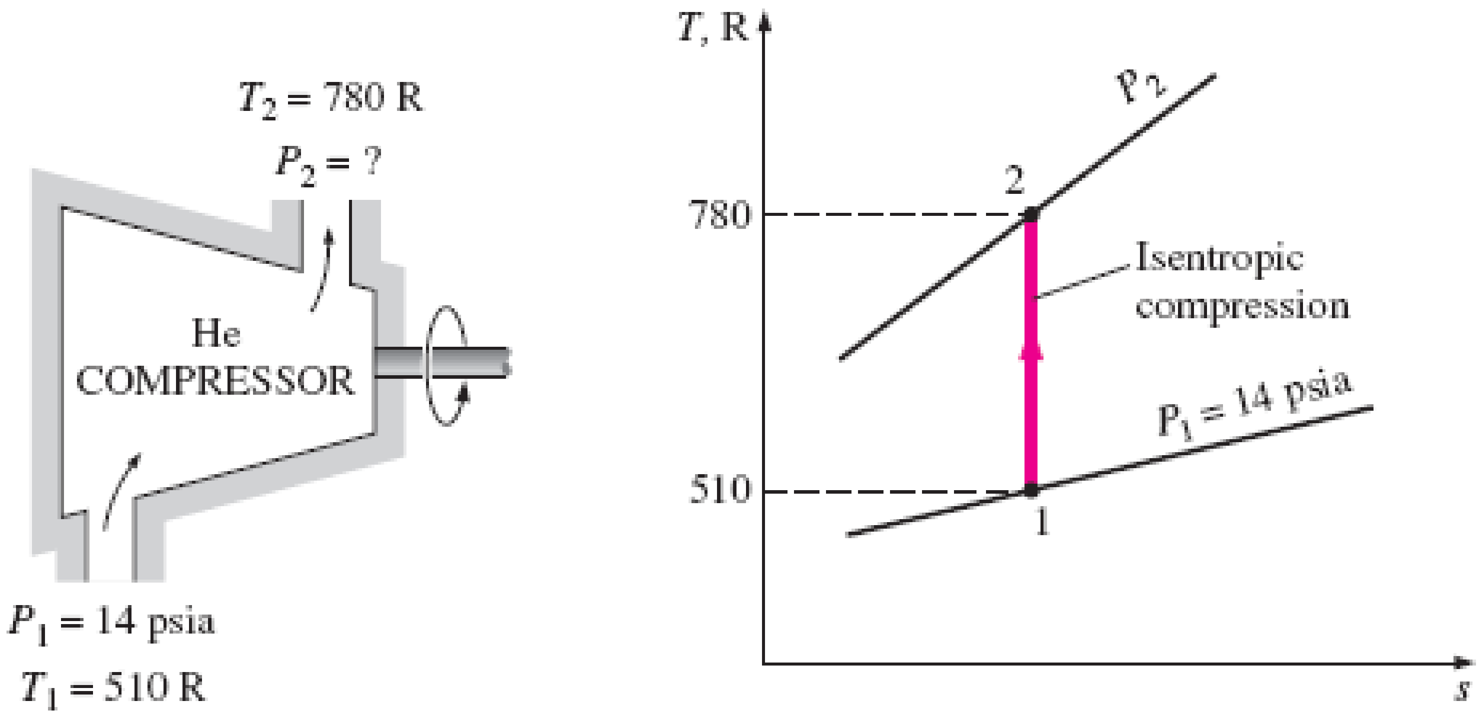
At $T_1 = 295$ K: $$v_{r1} = 647.9$$

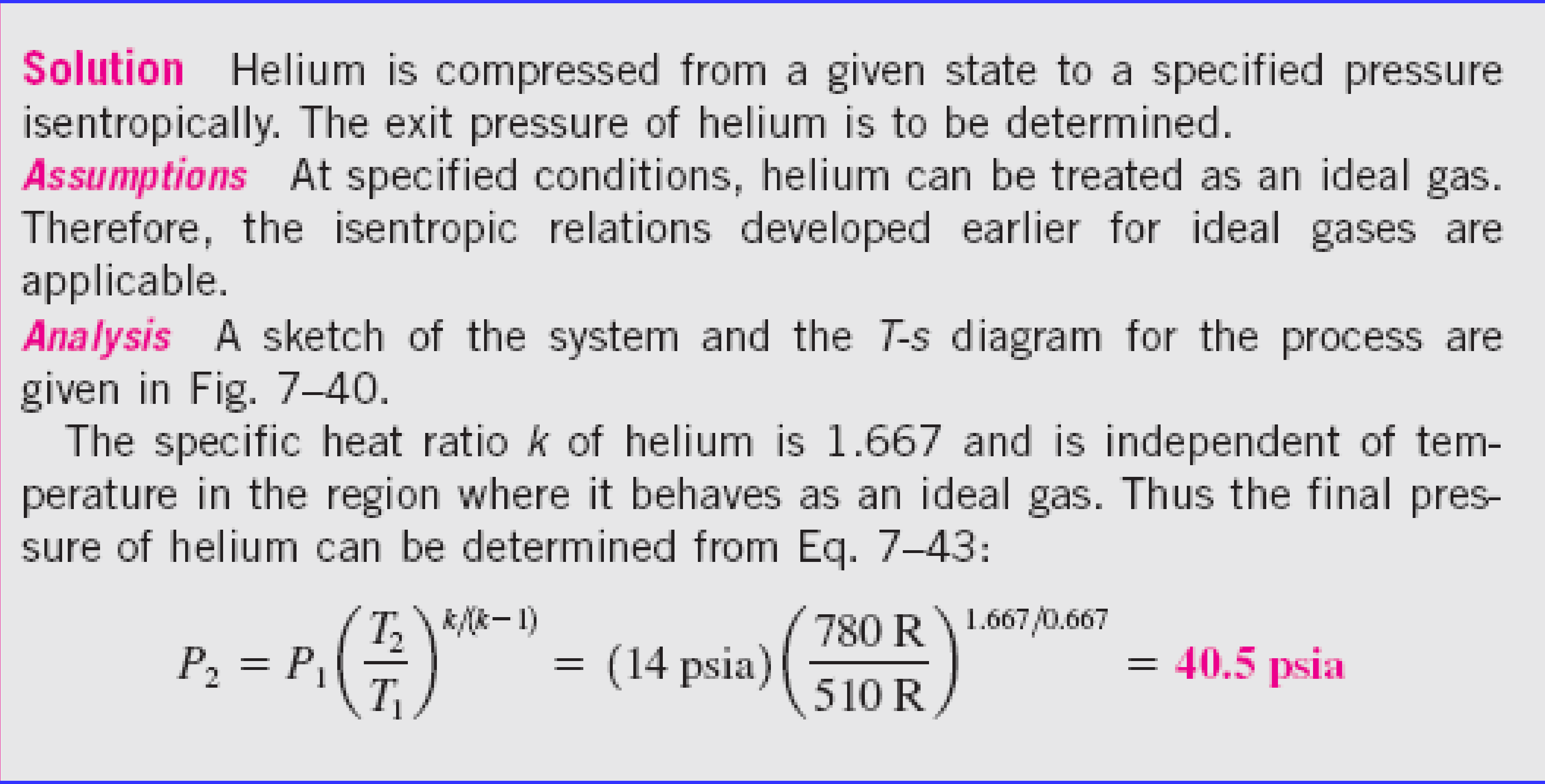
From Eq. 7–50: $$v_{r2} = v_{r1}\left(\frac{v_2}{v_1}\right) = (647.9)\left(\frac{1}{8}\right) = 80.99 \rightarrow T_2 = \mathbf{662.7\ K}$$

Therefore, the temperature of air will increase by 367.7°C during this process.

EXAMPLE 7–11 Isentropic Compression of an Ideal Gas

Helium gas is compressed by an adiabatic compressor from an initial state of 14 psia and 50°F to a final temperature of 320°F in a reversible manner. Determine the exit pressure of helium.





Solution Helium is compressed from a given state to a specified pressure isentropically. The exit pressure of helium is to be determined.

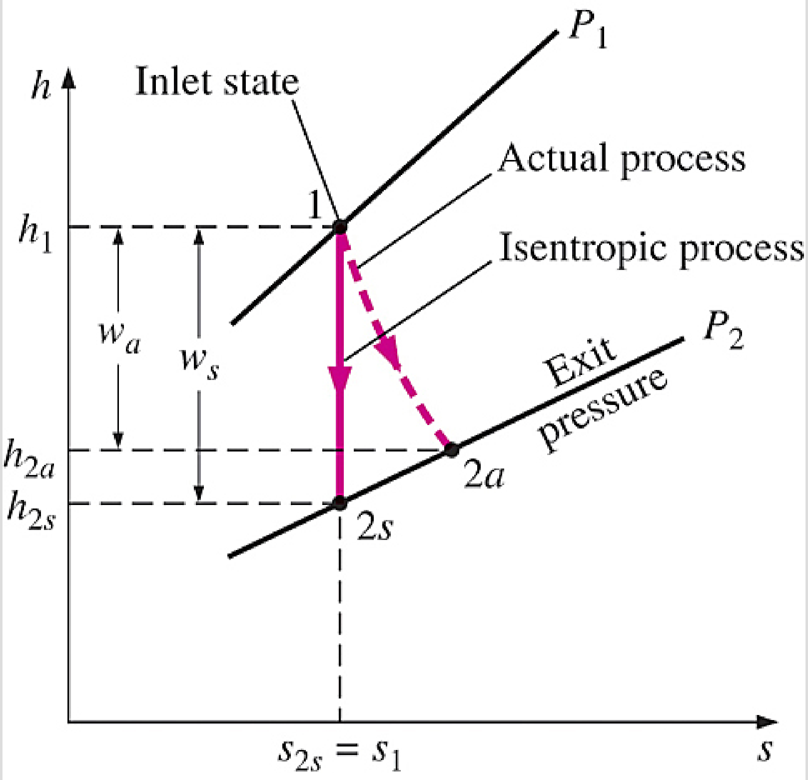
Assumptions At specified conditions, helium can be treated as an ideal gas. Therefore, the isentropic relations developed earlier for ideal gases are applicable.

Analysis A sketch of the system and the *T-s* diagram for the process are given in Fig. 7–40.

The specific heat ratio *k* of helium is 1.667 and is independent of temperature in the region where it behaves as an ideal gas. Thus the final pressure of helium can be determined from Eq. 7–43:

$$P_2 = P_1\left(\frac{T_2}{T_1}\right)^{k/(k-1)} = (14\text{ psia})\left(\frac{780\text{ R}}{510\text{ R}}\right)^{1.667/0.667} = \mathbf{40.5\text{ psia}}$$

Isentropic Efficiency of Turbines



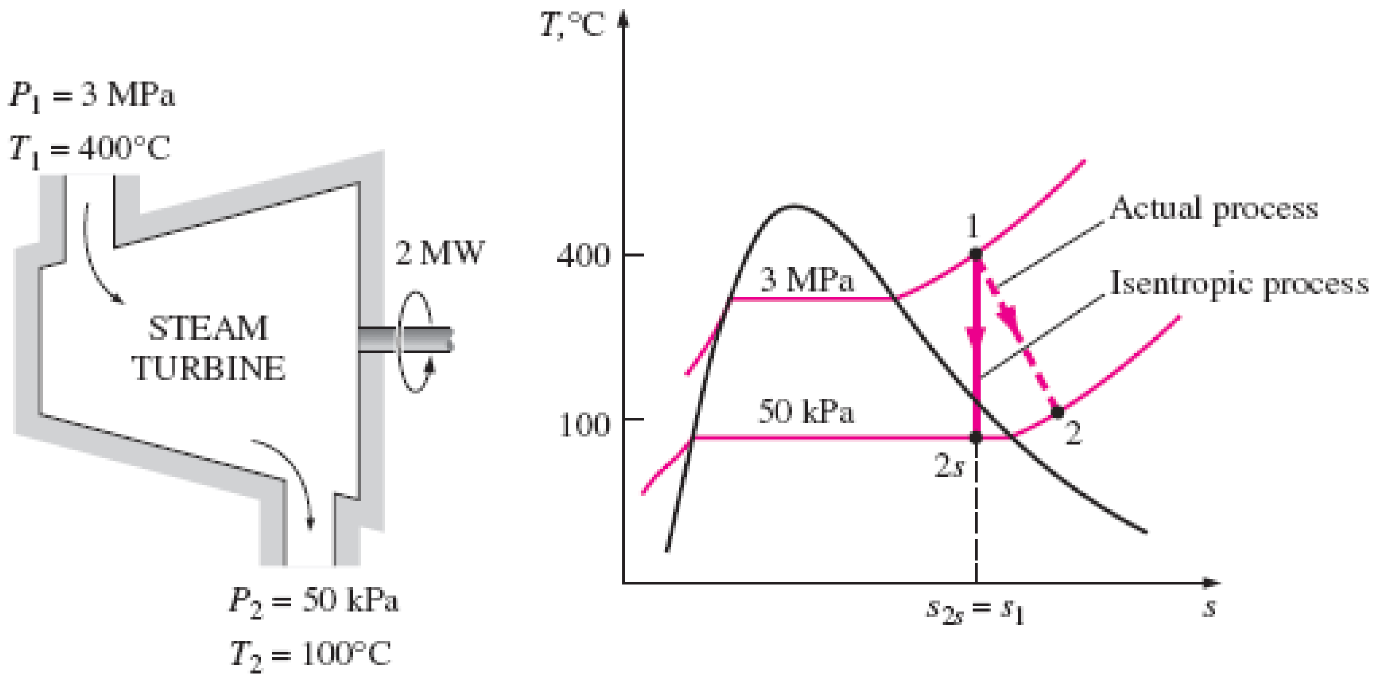
The *h-s* diagram for the actual and isentropic processes of an adiabatic turbine.

$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s}$$

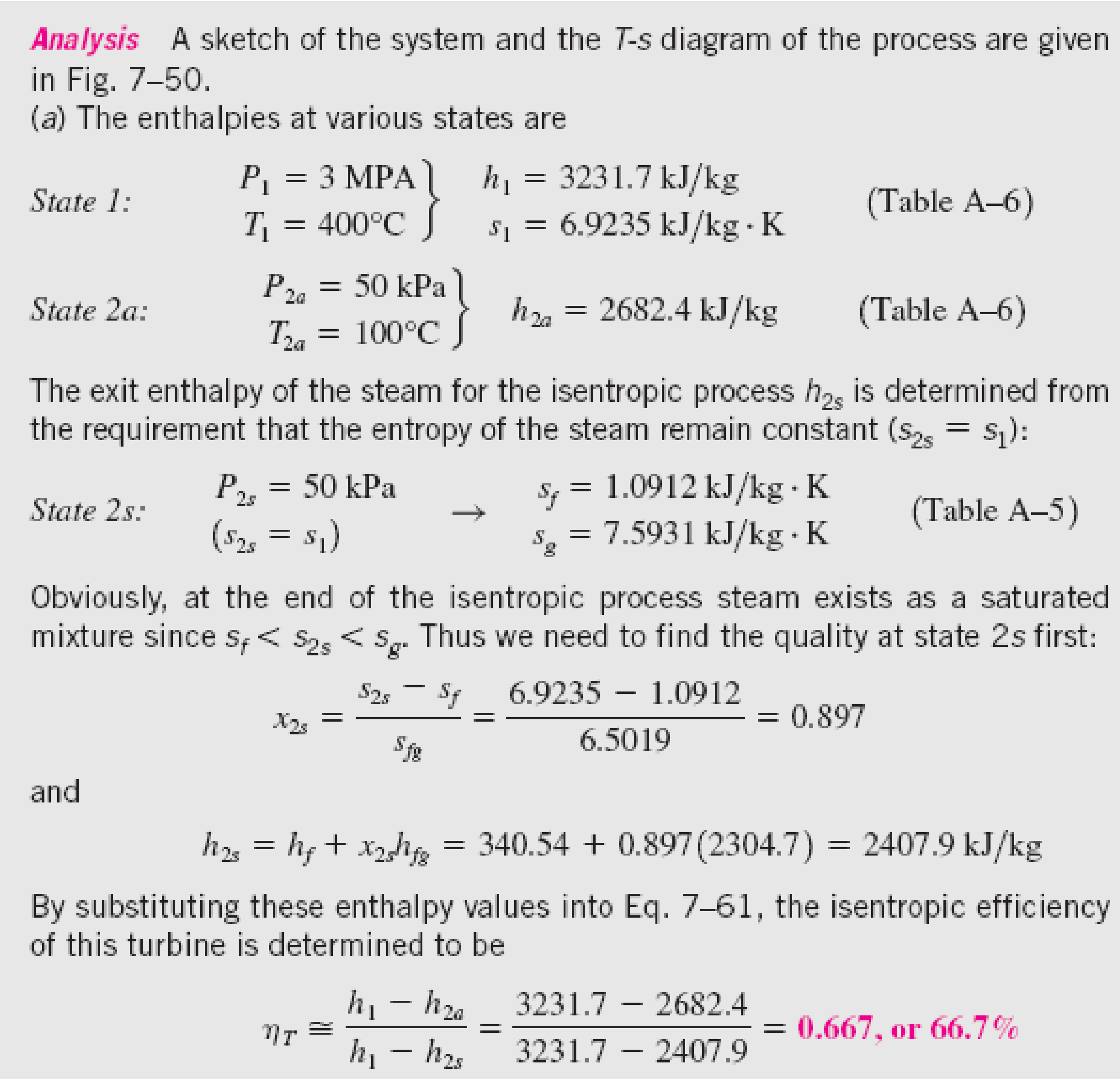
$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

EXAMPLE 7–14 Isentropic Efficiency of a Steam Turbine

Steam enters an adiabatic turbine steadily at 3 MPa and 400°C and leaves at 50 kPa and 100°C. If the power output of the turbine is 2 MW, determine (*a*) the isentropic efficiency of the turbine and (*b*) the mass flow rate of the steam flowing through the turbine.



$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$



Analysis A sketch of the system and the *T-s* diagram of the process are given in Fig. 7–50.

(*a*) The enthalpies at various states are

State 1:
$$\left.\begin{aligned} P_1 &= 3\text{ MPA} \\ T_1 &= 400^\circ\text{C} \end{aligned}\right\} \quad \begin{aligned} h_1 &= 3231.7\text{ kJ/kg} \\ s_1 &= 6.9235\text{ kJ/kg}\cdot\text{K} \end{aligned} \qquad \text{(Table A–6)}$$

State 2a:
$$\left.\begin{aligned} P_{2a} &= 50\text{ kPa} \\ T_{2a} &= 100^\circ\text{C} \end{aligned}\right\} \quad h_{2a} = 2682.4\text{ kJ/kg} \qquad \text{(Table A–6)}$$

The exit enthalpy of the steam for the isentropic process h_{2s} is determined from the requirement that the entropy of the steam remain constant ($s_{2s} = s_1$):

State 2s:
$$\begin{aligned} P_{2s} &= 50\text{ kPa} \\ (s_{2s} &= s_1) \end{aligned} \quad \longrightarrow \quad \begin{aligned} s_f &= 1.0912\text{ kJ/kg}\cdot\text{K} \\ s_g &= 7.5931\text{ kJ/kg}\cdot\text{K} \end{aligned} \qquad \text{(Table A–5)}$$

Obviously, at the end of the isentropic process steam exists as a saturated mixture since $s_f < s_{2s} < s_g$. Thus we need to find the quality at state 2*s* first:

$$x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{6.9235 - 1.0912}{6.5019} = 0.897$$

and

$$h_{2s} = h_f + x_{2s}h_{fg} = 340.54 + 0.897(2304.7) = 2407.9\text{ kJ/kg}$$

By substituting these enthalpy values into Eq. 7–61, the isentropic efficiency of this turbine is determined to be

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{3231.7 - 2682.4}{3231.7 - 2407.9} = \mathbf{0.667,\ or\ 66.7\%}$$

(*b*) The mass flow rate of steam through this turbine is determined from the energy balance for steady-flow systems:

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}h_1 = \dot{W}_{a,\text{out}} + \dot{m}h_{2a}$$

$$\dot{W}_{a,\text{out}} = \dot{m}(h_1 - h_{2a})$$

$$2\ \text{MW}\left(\frac{1000\ \text{kJ/s}}{1\ \text{MW}}\right) = \dot{m}(3231.7 - 2682.4)\ \text{kJ/kg}$$

$$\dot{m} = \mathbf{3.64\ kg/s}$$

Isentropic Efficiencies of Compressors and Pumps

$$\eta_C = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a}$$

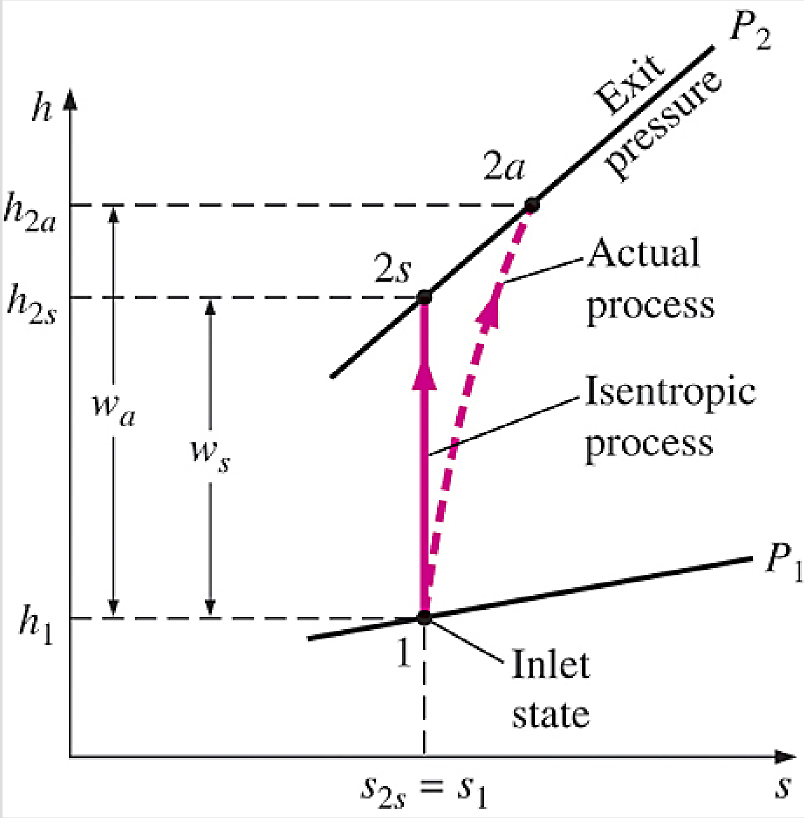
$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

For a pump

$$\eta_P = \frac{w_s}{w_a} = \frac{v(P_2 - P_1)}{h_{2a} - h_1}$$

Isothermal efficiency

$$\eta_C = \frac{w_t}{w_a}$$

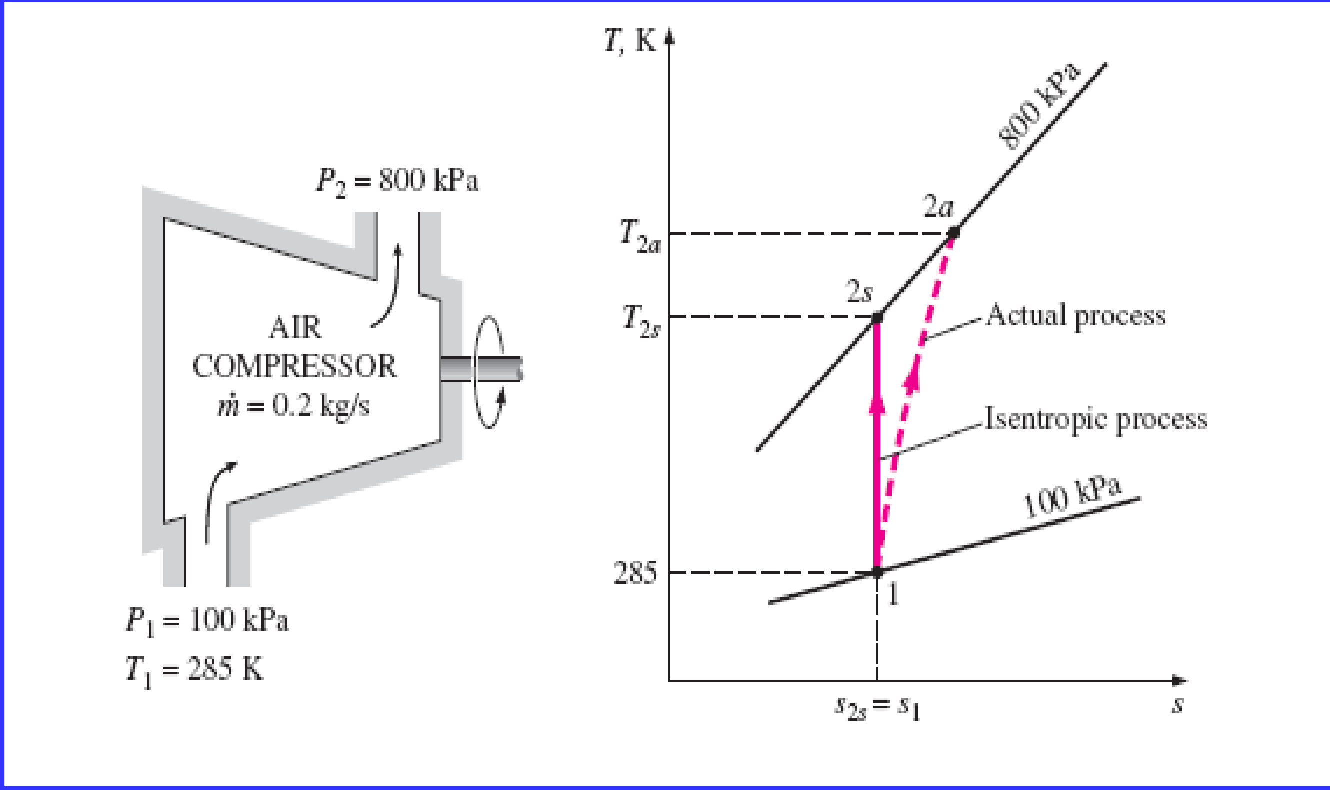


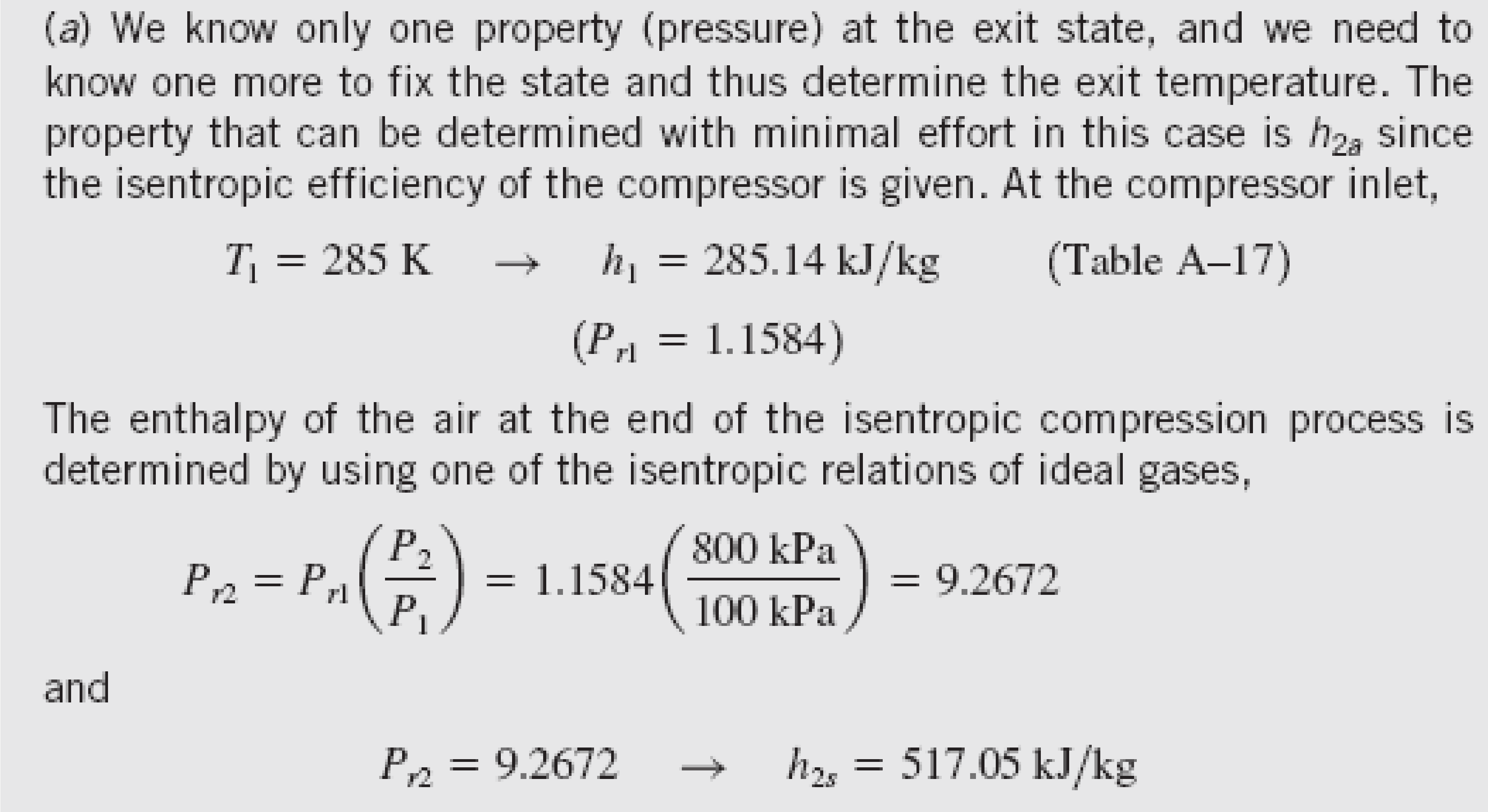
The *h*-*s* diagram of the actual and isentropic processes of an adiabatic compressor.

EXAMPLE 7–15 Effect of Efficiency on Compressor Power Input

Air is compressed by an adiabatic compressor from 100 kPa and 12°C to a pressure of 800 kPa at a steady rate of 0.2 kg/s. If the isentropic efficiency of the compressor is 80 percent, determine (*a*) the exit temperature of air and (*b*) the required power input to the compressor.

$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$





(*a*) We know only one property (pressure) at the exit state, and we need to know one more to fix the state and thus determine the exit temperature. The property that can be determined with minimal effort in this case is h_{2s} since the isentropic efficiency of the compressor is given. At the compressor inlet,

$$T_1 = 285 \text{ K} \quad \rightarrow \quad h_1 = 285.14 \text{ kJ/kg} \qquad \text{(Table A–17)}$$

$$(P_{r1} = 1.1584)$$

The enthalpy of the air at the end of the isentropic compression process is determined by using one of the isentropic relations of ideal gases,

$$P_{r2} = P_{r1}\left(\frac{P_2}{P_1}\right) = 1.1584\left(\frac{800 \text{ kPa}}{100 \text{ kPa}}\right) = 9.2672$$

and

$$P_{r2} = 9.2672 \quad \rightarrow \quad h_{2s} = 517.05 \text{ kJ/kg}$$

$$P_{r2} = P_{r1}\left(\frac{P_2}{P_1}\right) = 1.1584\left(\frac{800 \text{ kPa}}{100 \text{ kPa}}\right) = 9.2672$$

and

$$P_{r2} = 9.2672 \quad \longrightarrow \quad h_{2s} = 517.05 \text{ kJ/kg}$$

Substituting the known quantities into the isentropic efficiency relation, we have

$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \quad \longrightarrow \quad 0.80 = \frac{(517.05 - 285.14) \text{ kJ/kg}}{(h_{2a} - 285.14) \text{ kJ/kg}}$$

Thus,

$$h_{2a} = 575.03 \text{ kJ/kg} \quad \longrightarrow \quad T_{2a} = 569.5 \text{ K}$$

(*b*) The required power input to the compressor is determined from the energy balance for steady-flow devices,

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}h_1 + \dot{W}_{a,\text{in}} = \dot{m}h_{2a}$$

$$\dot{W}_{a,\text{in}} = \dot{m}(h_{2a} - h_1)$$

$$= (0.2\ \text{kg/s})[(575.03 - 285.14)\ \text{kJ/kg}]$$

$$= \mathbf{58.0\ kW}$$

Discussion Notice that in determining the power input to the compressor, we used h_{2a} instead of h_{2s} since h_{2a} is the actual enthalpy of the air as it exits the compressor. The quantity h_{2s} is a hypothetical enthalpy value that the air would have if the process were isentropic.

THE END